IIT JEE | MEDICAL | FOUNDATION

Solutions to Mock JEE Advanced-1 | 2024 | Paper-2

Mathematics

SINGLE CHOICE

1.(C) $x\int_0^x g(t)\,dt+\int_0^x (1-t)\,g(t)dt = x^4+x^2+ax+b$

Put $x = 0$ Both side we get $b = 0$

Now differentiate both side w.r.t. "x"

$$\int_0^x g(t)\,dt + xg(x)+(1-x)\,g(x) = 4x^3+2x+a$$

$$\Rightarrow\ g(x)+\int_0^x g(t)\,dt = 4x^3+2x+a$$

Put $x = 0$ both side we get $a = g(0)$

Now differentiate both side w.r.t x

$$g'(x)+g(x)=12x^2+2$$

Put $x = 0$ both side $\Rightarrow\ g'(0)+g(0)=2$

Now, $I_0=\int_0^1 \frac{6(a+g'(0))\,dx}{g'(x)+g(x)+b+10}=\int_0^1 \frac{6(2)\,dx}{12x^2+2+0+10}$

$$=\int_0^1 \frac{dx}{1+x^2}=\left[\tan^{-1}x\right]_0^1=\frac{\pi}{4}-0=\frac{\pi}{4}$$

2.(B) Clearly at 6th, 7th and 8th toss there must be tail, head, head respectively and in first five tosses no any two consecutive heads obtained, for first five tosses

Case-I all 5 "T" $\Rightarrow \frac{\lfloor 5}{\lfloor 5}=1\,way$

Case-II 4 "T" $\Rightarrow \frac{\lfloor 5}{\lfloor 4\lfloor 1}=5\,way$

Case-III 3 "T", 2 "H" $\_T\_T\_T\_$ $\Rightarrow\ {}^4C_2=6\ ways$ using gap method

Case-IV 2 "T", 3 "H" $\_T\_T\_$ $\Rightarrow\ {}^3C_3=1\ way$

$$\Rightarrow\ P(E)=\frac{1+5+6+1}{2^8}=\frac{13}{256}$$

3.(A) $f(x) = \tan^{-1}\left(\frac{|x|}{\sqrt{1-x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Domain of $f(x)$ is $x \in (-1,1)$

$\therefore\ f(x) = \sin^{-1}|x| + \tan^{-1}|x|$ is even function

Since, $\sin^{-1} x + \tan^{-1} x$ is increasing

Function in $[0,1) \Rightarrow\ f(0) \le f(x) < f(1)$

$\therefore\ f(x) \in \left[0, \frac{3\pi}{4}\right)$

Integers in range $= \{0,1,2\}$

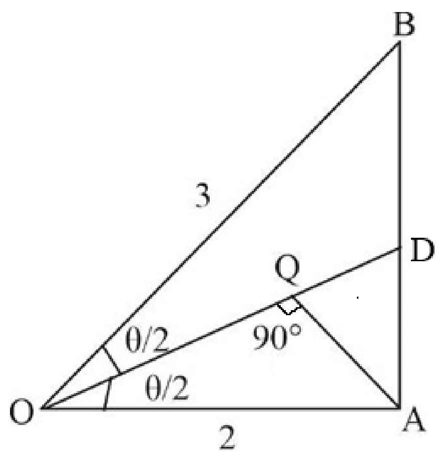
$f(x) = 2$ has two solutions

$\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ $\quad \therefore\ \sin\theta > \cos\theta\ ;\ f(\sin\theta) > f(\cos\theta)$

4.(B) $\cos\theta = \frac{4+9-(AB)^2}{2(2)(3)} \Rightarrow (AB)^2 = 13 - 12(2\cos^2\frac{\theta}{2} - 1)$

$(AB)^2 = 25 - 24\cos^2\frac{\theta}{2}$

$AD = \frac{2}{5}AB,\ AQ = OA\sin\frac{\theta}{2}$



$(QD)^2 = (AD)^2 - (AQ)^2 = \frac{4}{25}(25 - 24\cos^2\frac{\theta}{2}) - 4\sin^2\frac{\theta}{2}$

$= 4 - \frac{96}{25}\cos^2\frac{\theta}{2} - 4\sin^2\frac{\theta}{2} = 4\cos^2\frac{\theta}{2} - \frac{96}{25}\cos^2\frac{\theta}{2} = \frac{4}{25}\cos^2\frac{\theta}{2}$ D

$QD = \frac{2}{5}\cdot\frac{1}{5} \Rightarrow\ 25QD = 2$

ONE OR MORE THAN ONE CHOICE

5.(ABCD) $A+B=ABAB=A^2+A \Rightarrow B=A^2$

$BAB=A^5=A+I \Rightarrow A(A^4-I)=I$

$B^5-A^5=(A^5)^2-A^5=(A+I)^2-(A+I)=A^2+A=A+B$

$B^5-A^5=A+B \Rightarrow A^{10}-A^5=A+A^2 \Rightarrow A^9-A^4=I+A$

$A^2B^2=A^2(A^2)^2=A^6$ also, $BA^2B=A^2A^2A^2=A^6 \Rightarrow A^2B^2=BA^2B$

6.(BD) $\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}\dfrac{P\left(\dfrac{1}{x^3}\right)}{e^{\frac{1}{x^4}}}=\lim\limits_{t\to\infty}\dfrac{P(t^3)}{e^{t^4}}=0$

As we know $\lim\limits_{x\to\infty}\dfrac{x^n}{e^x}=0, n\in N \Rightarrow \lim\limits_{x\to 0} f(x)=f(0)=0$

$f'(0)=\lim\limits_{h\to 0}\dfrac{P\left(\dfrac{1}{h^3}\right)}{he^{\frac{1}{h^4}}}=\lim\limits_{t\to\infty}\dfrac{tP\left(t^3\right)}{e^{t^4}}=0$

7.(ACD) $f(x+y)=g(x)+h(y)$ ...(i)

Put $x=0$

$f(y)=g(0)+h(y) \Rightarrow h(y)=f(y)-g(0)$ ...(ii)

Put $y=0$

$f(x)=g(x)+h(0) \Rightarrow g(x)=f(x)-h(0)$ ...(iii)

Put (ii) and (iii) in (i)

$f(x+y)=f(y)+f(x)-h(0)-g(0)$

Define $C_1(x)=f(x)-h(0)-g(0)\forall x\in R$

$\Rightarrow C_1(x+y)=C_1(x)+C_1(y)$

The solution of this functional equation is obtained by differentiation through first principles as $C_1(x)=cx$ (where c is a constant)

$\Rightarrow f(x)=C_1(x)+h(0)+g(0)=cx+h(0)+g(0)$

$g(x)=f(x)-h(0)=cx+g(0)$

$h(x)=f(x)-g(0)=cx+h(0)$

$f'(0)=f'(1)=c$

NON-NEGATIVEINTEGER TYPE

1.(2) $\dfrac{3x^2+1}{\sqrt{x^4+x^2}}=\dfrac{3x^2+1}{x\sqrt{x^2+1}}=\dfrac{x^2+1+2x^2}{x\sqrt{x^2+1}}$

$$=\frac{\sqrt{x^2+1}}{x}+\frac{2x}{\sqrt{x^2+1}}\geq 2\sqrt{2}$$

2.(6) $-e^{-x^2}\dfrac{dy}{dx}=2xy^2$

$$\int\frac{dy}{y^2}=\int -2xe^{x^2}dx$$

$$-\frac{1}{y}=-e^{x^2}-c$$

$$\frac{1}{y}=e^{x^2}+c \Rightarrow \ y=\frac{1}{e^{x^2}+c}=f(x)$$

$$f(0)=\frac{1}{2}=\frac{1}{1+c}\Rightarrow \ c=1$$

$$\Rightarrow \ f(x)=\frac{1}{e^{x^2}+1}$$

Since, $0<f(x)\leq\dfrac{1}{2}$

3.(7) Let E_1 : first ball drawn red from Urn A and 2nd ball drawn black from B

E_2 : first ball drawn white from A and 2nd ball drawn black from B

E_3 : first ball drawn red from B and 2nd ball drawn black from B

E_4 : first ball drawn black from B and 2nd ball drawn black from B

$$P=\frac{P(E_1)+P(E_3)}{P(E_1)+P(E_2)+P(E_3)+P(E_4)}$$

$$P(E_1)=\left(\frac{1}{2}\right)\left(\frac{2}{6}\right)\left(\frac{1}{2}\right)\left(\frac{3}{6}\right)=\frac{6}{144}$$

$$P(E_2)=\left(\frac{1}{2}\right)\left(\frac{4}{6}\right)\left(\frac{1}{2}\right)\left(\frac{3}{6}\right)=\frac{12}{144}$$

$$P(E_3)=\left(\frac{1}{2}\right)\left(\frac{3}{6}\right)\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)=\frac{9}{120}$$

$$P(E_4)=\left(\frac{1}{2}\right)\left(\frac{3}{6}\right)\left(\frac{1}{2}\right)\left(\frac{2}{5}\right)=\frac{6}{120}$$

$$\Rightarrow \ P=\frac{7}{15}$$

4.(6) $A^n = 1$

$\Rightarrow \; A = (1)^{1/n} = e^{2\pi ri/n}, \; r = 0,1,2,....n-1$

$\therefore \; A = 1, e^{2\pi i/n}, e^{4\pi i/n}, e^{6\pi i/n}, ..., e^{2\pi(n-1)i/n}$

$(A+1)^n = 1 \Rightarrow A+1 = (1)^{1/n} = e^{2\pi pi/n}$

$\Rightarrow \; A = e^{2\pi pi/n} - 1 = e^{p\pi i/n} 2i\sin\left(\frac{\pi p}{n}\right)$

$p = 0,1,2,....,n-1$

$\therefore \; A = 0, e^{\pi i/n} 2i\sin\left(\frac{\pi}{n}\right), e^{2\pi i/n} 2i\sin\left(\frac{2\pi}{n}\right), ..., e^{\pi i(n-1)/n} 2i\sin\left(\frac{\pi(n-1)}{n}\right)$

$n = 6$

$e^{4\pi i/n} = e^{4\pi i/6} = e^{2\pi i/3}$

$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$

$e^{\pi i/n} 2i\sin\left(\frac{\pi}{n}\right) = e^{\pi i/6} 2i\sin\left(\frac{\pi}{6}\right) = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)i$

$= \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)i = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$

Hence, the least value of n is 6.

5.(0) $B = A^{2^n} = A^{2\cdot 2^{n-1}} = (A^2)^{2^{n-1}} = (A^{-1})^{2^{n-1}} \; \left[\because A^2 = A^{-1}\right]$

$= (A^{2^{n-1}})^{-1} = (A^{2\cdot 2^{n-2}})^{-1} = \left[(A^2)^{2^{n-2}}\right]^{-1}$

$= \left[(A^{-1})^{2^{n-2}}\right]^{-1} = ((A^{-1})^{-1})^{2^{n-2}} = A^{2^{n-2}} = C$

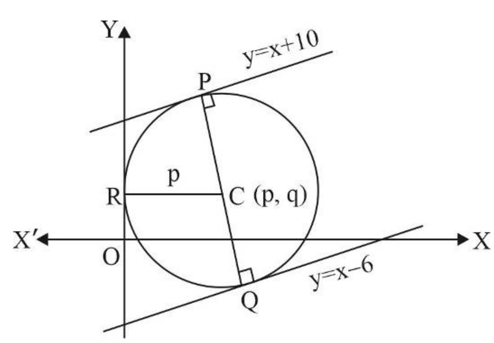
$\Rightarrow \; B - C = 0 \Rightarrow \det(B-C) = 0$

6.(6) $CP = CR$

$\Rightarrow \; \frac{|p-q+10|}{\sqrt{2}} = p$

$p - q + 10 = p\sqrt{2}$ …(i)

$\frac{p-q+10}{\sqrt{2}} = -\left(\frac{p-q-6}{\sqrt{2}}\right)$ *or* $p - q = -2$ …(ii)



From Eqs. (i) and (ii), we get

$p = 4\sqrt{2}$ & $q = 4\sqrt{2} + 2$

Now, $p + q = 2 + 8\sqrt{2} = a + b\sqrt{2}$ (given)

Hence, $|a-b| = |2-8| = 6$

COMPREHENSION WITH NUMERICAL TYPE

7.(6) $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9=\sum_{r=0}^{9}{}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

For the term that is independent of *x*,

We must have $18-2r-r=0 \quad \Rightarrow \quad r=6$

Required term $={}^9C_6\left(\frac{3}{2}x^2\right)^3\left(-\frac{1}{3x}\right)^6={}^9C_6\left(\frac{1}{6}\right)^3$

8.(0) Let $(r+1)^{\text{th}}$ term of $\left(\frac{5}{x^2}+x^4\right)^n$ be independent of *x*, we have

$$T_{r+1}={}^nC_r\left(\frac{5}{x^2}\right)^{n-r}(x^4)^r={}^nC_r\,5^{n-r}x^{6r-2n}$$

For this term to be independent of x, $6r-2n=0$ or $n=3r$

$\therefore$ Each of 18, 21, 27, 99 is divisible by 3.

9.(6) $S=\{1,2,3,4,5,.....,21\}$

Total number of ways choosing *x* and *y* is ${}^{21}C_2=\frac{21\cdot 20}{1\cdot 2}=210$

Now, arrange the given numbers as below :

1	4	7	10	13	16	19
2	5	8	11	14	17	20
3	6	9	12	15	18	21

We see that, $x^3+y^3=(x+y)(x^2-xy+y^2)$ will be divisible by 3 in the following cases :

One of two numbers belongs to the first row and one of the two numbers belongs to the second row or both numbers occurs in third row,

Number of favourable cases $=({}^7C_1)({}^7C_1)+{}^7C_2=70$

Required probability $=\frac{70}{210}=\frac{1}{3}$

10.(10) Given, x,y,z are in AP

$2y=x+z$

It is clear that sum of *x* and *z* is even.

$\therefore$ *x* and *z* both are even or odd out of set *S*.

i.e., 11 numbers (1, 3, 5,….,21) are odd and 10 numbers (2, 4, 6,….,20) are even

Number of favourable cases $={}^{11}C_2+{}^{10}C_2=\frac{11\cdot 10}{1\cdot 2}+\frac{10\cdot 9}{1\cdot 2}=100$

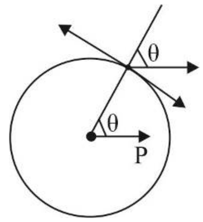
and total number of ways choosing x,y and *z* is ${}^{21}C_3=\frac{21\cdot 20\cdot 19}{1\cdot 2\cdot 3}=1330$

Required probability $=\frac{100}{1330}=\frac{10}{133}$

Physics

SINGLE CHOICE

1.(D) $\frac{\sigma}{2\varepsilon_0}\sin\theta = \frac{KP\sin\theta}{R^3}$



$\sigma = \frac{P}{2\pi R^3}$

2.(C) $[F] = [MLT^{-2}]$

$[\rho] = [ML^{-3}T^0]$

$[V] = [LT^{-1}]$

$[A] = [L^2]$

$[F] = [\rho]^a[V]^b[A]^c$

$\Rightarrow\ MLT^{-2} = [ML^{-3}]^a[LT^{-1}]^b[L^2]^c \ \Rightarrow\ MLT^{-2} = M^aL^{-3a+b+2c}T^{-b} = M^aL^{-3a+2c}T^{-b}$

$a=1,\ b=2,\ -3a+b+2c=1$

$\Rightarrow\ -3+2+2c=1 \ \Rightarrow\ 2c=2 \ \Rightarrow\ c=1$

3.(B) At any time t let $\vec{V}$ be velocity of boat

$\vec{V} = (V_x)\hat{x} + (V_y)\hat{y}$

$\vec{V} = \left[\frac{u_0}{d^2}y(d-y)\right]\hat{x} + (kt)\hat{y}$ …(i)

From equation (i) and (ii)

$y = 0 + \frac{1}{2}kt^2$ …(ii)

$\vec{V} = \left[\frac{u_0}{d^2}\frac{kt^2}{2}\left(d - \frac{kt^2}{2}\right)\right]\hat{x} + (kt)\,\hat{y}$

$\vec{a} = \frac{d\vec{V}}{dt} = \frac{u_0k}{2d^2}\left[2dt - \frac{k}{2}(4t^3)\right]\hat{x} + (k)\,\hat{y}$ …(iii)

Time t_0, at which $y = \frac{d}{2}$

$\Rightarrow\ \frac{1}{2}kt_0^2 = \frac{d}{2}$; $t_0 = \sqrt{\frac{d}{k}}$; $\vec{a} = \frac{u_0k}{2d^2}\left[2d\sqrt{\frac{d}{k}} - 2k\frac{d}{k}\sqrt{\frac{d}{k}}\right]\hat{x} + k\hat{y}$; $\vec{a} = k\hat{y}$

4.(B) $V_{rms} = \sqrt{\frac{3RT}{M}} \propto \sqrt{T}$

If $V_{rms} \to 2V_{rms}$

$T \to 4T$

Final temp $= 4\times300 = 1200K$

$\Delta T_2 = (900K)$

At constant volume

$\Delta Q = \Delta u = nC_V\Delta T = \left(\frac{14}{28}\right)\left(\frac{5R}{2}\right)\times900 = \frac{14}{28}\times\frac{5}{2}\times\frac{28}{3.5}\times900 = 9kJ$

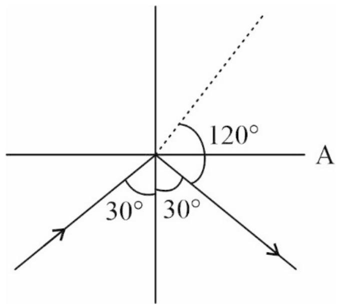
ONE OR MORE THAN ONE CHOICE

5.(AD) For T.I.R. at A

$4\sin 30^\circ = \mu_2 \sin 90^\circ$

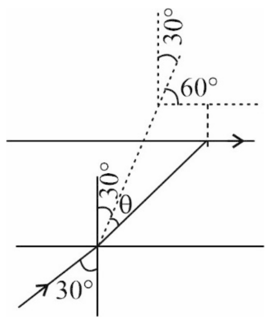
$\mu_2 = 2$

Case-I



If $\mu_2 < 2,$ then always T.I.R. takes place and in this situation angle of deviation is 120°.

Case-II



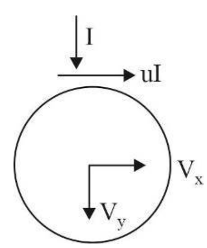
$\mu_2 > 2,$ then for the given angle of incidence no T.I.R. take place at A so light strike on the interface B for T.I.R.

$\mu_2 \sin\alpha = 2\sin 90^\circ$ or $\mu_2 = \dfrac{2}{\sin\alpha}$

$\sin\alpha < 1$; $\mu_2 > 2$

6.(BD) Let ball attained speed V & angular speed ω about centre of mass

$I_0 = mV$



$V = \dfrac{40}{2} = 20\,m/s$ and $I_0 r = \dfrac{mR^2}{2}\omega$

$\omega = \dfrac{40\times 10^{-2}\times 2}{2\times 4\times 10^{-4}}$

$\omega = 10\times 10^{-2}\times 10^4 = 1000\,rad/s$

$I_0 =$ moment of inertia about COM $= \dfrac{mR^2}{2} = \dfrac{2}{2}\times (2)^2\times 10^{-4} = 4\times 10^{-4}\,kg\cdot m^2$

$L_i =$ Angular momentum about COM before collision

$= I_0\omega = 4\times 10^{-4}\times 10^3 = 0.4\,kg\dfrac{m^2}{S}$

After collision

$\vec{V}_f = (V_x)\hat{i} - (V_y)\hat{j}$

$V_y = 20\, m/s$

$$V_x = \frac{\mu I}{m} = \frac{\left(\frac{1}{4}\right)}{2} \times m(V_y + V) = \frac{1}{8} \times 2 \times (40)$$

$V_x = 10\, m/s$

Angular impulse $= (\mu IR) = \frac{1}{4} \times 2 \times 40 \times 2 \times 10^{-2} = 0.4\, kg \frac{m^2}{s}$

$\vec{L}_f - \vec{L}_i = -0.4$

$\Rightarrow\ \vec{L}_f = 0$

So, disc stops rotational motion about COM

Total no. of rotations $= \frac{\omega(\Delta t)}{2\pi} = \frac{\omega \times \frac{20}{20}}{2\pi} = \frac{1000}{2\pi} = \left(\frac{500}{\pi}\right)$

and total distance travelled by COM before 2nd collision $= (20 m/s \times 1\text{sec}) + \left(\sqrt{20^2 + 10^2}\right) \times 1\text{sec}$

$$= 20 + 10\sqrt{5} = 10(2 + \sqrt{5})m$$

7.(ABCD) The energy consumed every second by a 1000 W bulb = 1000 J

As the working efficiency of the bulb is equal to 2.5% the energy radiated by the bulb per second

$$\Delta U = 1000 \times \frac{2.5}{100}$$

$$\therefore\ \Delta U = 25\, Js^{-1}$$

Consider the bulb at the centre of the sphere, surface area of the sphere

$A = 4\pi R^2 = (4)(3.14)(10^2) = 1256\, m^2$

Intensity I $\quad I = \frac{Energy}{(time)(area)} = \frac{25}{1256} = 0.02 Wm^{-2}$

$$\therefore\ E_{rms} = \left[\frac{0.02}{8.85 \times 10^{-12} \times 3.0 \times 10^8}\right]^{1/2} = 2.74\, Vm^{-1}$$

$$B_{rns} = \frac{E_{rms}}{c}$$

$$B_{rms} = \frac{2.74}{3.0 \times 10^8} = 9.13 \times 10^{-9}\, T$$

$$E_0 = \sqrt{2} E_{rms}$$

$$E_0 = 1.41 \times 2.74 = 3.86\, Vm^{-1}$$

$$B_0 = \sqrt{2} B_{rms}$$

$$B_0 = 1.41 \times 9.13 \times 10^{-9} = 1.29 \times 10^{-8}\, T$$

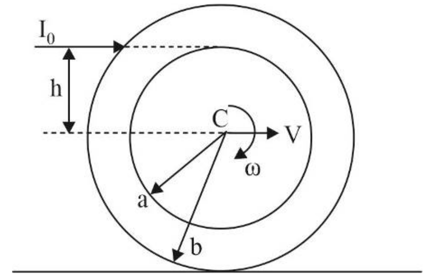
The total energy incident on the surface = 25J

$\therefore$ The moment (ΔP) imparted to the surface in one second (= force)

$$\Delta P = \frac{\Delta U}{c} = F = \frac{25}{3 \times 10^8} = 8.33 \times 10^{-8}\, N$$

NON-NEGATIVEINTEGER TYPE

1.(2) In case of pure rolling friction force is zero



$v = \omega b$ ...(i)

$I_0 = mv = m\omega b$

$\omega = \frac{I_0}{mb}$ ...(ii)

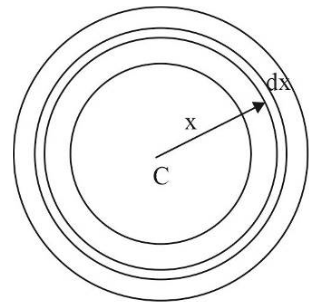
From angular impulse equation

$I_0 h = I_c \omega$ ...(iii) [$I_c =$ moment of inertia abut COM]

From equation (ii) & (iii)

$I_0 h = I_c \cdot \frac{I_0}{mb}$

$h = \frac{I_c}{mb}$ ...(iv)



Let's calculate moment of inertia

$\rho = density = \frac{m}{\frac{4}{3}\pi\left[b^3 - a^3\right]} = \frac{3m}{4\pi(b^3 - a^3)}$

$dI = \frac{2}{3}(dm)x^2$

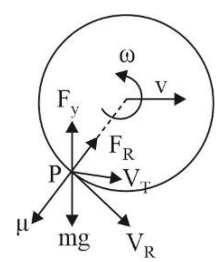
$I_c = \int_c^b \left(\frac{2}{3}\rho 4\pi x^2 dx\right)x^2$; $I_c = \frac{8}{3}\rho\pi\int_a^b x^4 dx = \frac{8}{3}\rho\pi\frac{(b^5 - a^5)}{5}$

$I_c = \left(\frac{8}{3}\pi\right)\frac{3m}{4\pi(b^3 - a^2)}\frac{(b^5 - a^5)}{5} = \frac{2m}{5}\frac{(b^5 - a^5)}{(b^3 - a^3)}$ ...(v)

From equation (iv) and (v)

$h = \frac{I_c}{mb} = \frac{2}{5b}\frac{(b^5 - a^5)}{(b^3 - a^3)}$

2.(1) Velocity of point P will be $\vec{V}_P = \vec{V}_T + \vec{V}_R$;



$\vec{V}_T$: Translational motion velocity, $V_T = V$

$\vec{V}_R$: Velocity done to rotation ; $V_R = r\omega = V$

Magnetic force, $\vec{F}_B = q(\vec{V}_P \times \vec{B})$

$= q(\vec{V}_T + \vec{V}_R) \times \vec{B} = q(\vec{V}_T \times \vec{B}) + q(\vec{V}_R \times \vec{B})$

$\vec{F}_B = \vec{F}_T + \vec{F}_R$

For no force of interaction between the ring and the particle, N = 0

$\Rightarrow$ F_R will provide necessary centripetal force requirement for particle to rotate on a circle

$\Rightarrow F_R = \frac{mv^2}{r} \Rightarrow qV_R B = \frac{mv^2}{r} \Rightarrow qvB = \frac{mv^2}{r} \Rightarrow qB = \frac{mv}{r}$ ...(i)

To ensure $N = 0$, $F_y = mg \Rightarrow qvB = mg$ ...(ii)

Using (i) and (ii), $\frac{mg}{v} = \frac{mv}{r} \Rightarrow r = \frac{v^2}{g}$ and $q = \frac{mg}{vB}$

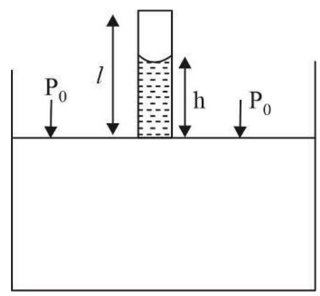
3.(2) At a distance x from end A, temp is

$$T = T_1 + \left(\frac{T_1 - T_2}{l}\right)x \qquad T_1 = 225K,\ T_2 = 256\,K$$

$$V = 10\sqrt{T} \quad \Rightarrow \frac{dx}{dt} = 10\sqrt{T}$$

$$\int \frac{dx}{\sqrt{T}} = \int_0^t 10dt \ ; \qquad t = \frac{2l}{10\left(\sqrt{T_1} + \sqrt{T_2}\right)} = \frac{2\times 310}{10(15+16)} = 2\sec$$

4.(2) Apply Boyle's law



$$P_1V_1 = P_2V_2$$

$$P_0Al = \left(P_0 - \rho gh + \frac{4T}{d}\cos\theta\right)A(l-h)$$

$$\Rightarrow \frac{4T}{d}\cos\theta = P_0\frac{l}{l-h} - P_0 + \rho gh$$

$$\Rightarrow \frac{4T}{d}\cos\theta = P_0\left(\frac{l-l+h}{l-h}\right) + \rho gh$$

$$\Rightarrow T = \left(\frac{d}{4\cos\theta}\right)\left[P_0\frac{h}{l-h} + \rho gh\right] \quad \Rightarrow T = \frac{d}{4\cos\theta}\left[P_0\frac{l/2}{l/2} + P_0\right] \quad \Rightarrow T = \left(\frac{P_0 d}{2\cos\theta}\right)$$

5.(30) $A = A_0e^{-\lambda t} \ \Rightarrow \ \frac{6}{100} = e^{-\frac{\ln 2}{T_{\frac{1}{2}}}}$

$$\frac{1}{2^{\frac{t}{T_{\frac{1}{2}}}}} = \frac{6}{100} \ \Rightarrow \ \frac{1}{2^4} \ ; \ T_{\frac{1}{2}} = 30 \text{ minutes}$$

6.(6) $i = \frac{6}{R+2r} = 1$

$R + 2r = 6$

$R = 6 - 2r$ …(1)

Also $i = \frac{3}{R + r/2} = 0.6$

$R + \frac{r}{2} = 5$ …(2)

From equation (1) and (2)

$$6 - 2r + \frac{r}{2} = 5$$

$$\frac{3r}{2} = 1$$

$$r = \frac{2}{3}$$

COMPREHENSION WITH NUMERICAL TYPE

7.(0.92), 8.(56)

Initially capacitors act as conducting wire. So initially all the three resistors will be in parallel.

$$R_{eq} = \frac{12}{13} = 0.92$$

After long time, resistors will be in series, $R_{eq} = 2 + 3 + 4 = 9\Omega$

$$I = \frac{24}{9} = \frac{8}{3} A$$

Potential difference $3\mu F =$ potential difference across $(3\Omega + 4\Omega)$

$$\Rightarrow V_{3\mu F} = (3+4)I = \frac{56}{3} V$$

$$q = CV_{3\mu F} = \frac{3 \times 56}{3} = 56\mu C$$

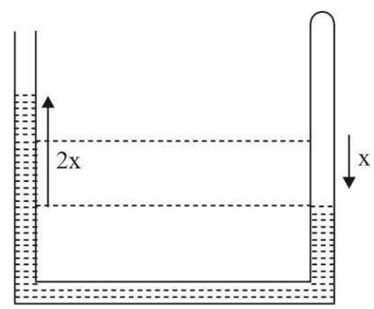
9.(5) & 10.(7)

$$w = w_1 + w_2 + w_3$$

$w_1 =$ work done by gas till right vertical limb is filled with gas

$w_1 + w_2 =$ work done by gas till right vertical limb is filled and horizontal limb is filled with gas

$$w_1 = \int_{x=0}^{l/2} (P_0 + \rho g(2x)Adx = P_0 \frac{Al}{2} + \frac{\rho g l^2}{4} \cdot A \quad \ldots\text{(i)}$$



$$w_2 = (P_0 + \rho g l) A \int_0^l dx = P_0 Al + \rho g A l^2 \quad \ldots\text{(ii)}$$

$$w_3 = \int_0^l (P_0 + \rho g(l - y))\, dy\, A = P_0 Al + \rho g A l^2 - \frac{\rho g A}{2} l^2 \quad \ldots\text{(iii)}$$

$$W = \left(\frac{5P_0 Al}{2}\right) + \left(2 - \frac{1}{4}\right) \rho g A l^2$$

$$= \underbrace{\frac{5P_0 \rho l}{2}}_{\text{work done against atmos-pheric pressure}} + \underbrace{\frac{7}{4} \rho g A l^2}_{\text{work done against gravity}}$$

Chemistry

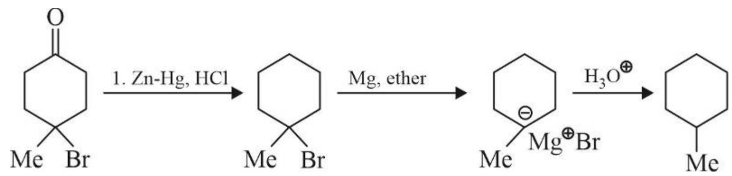
SINGLE CHOICE

1.(A) CN^+ has 12 electrons (6 from carbon, 7 from nitrogen, -1 for positive charge)

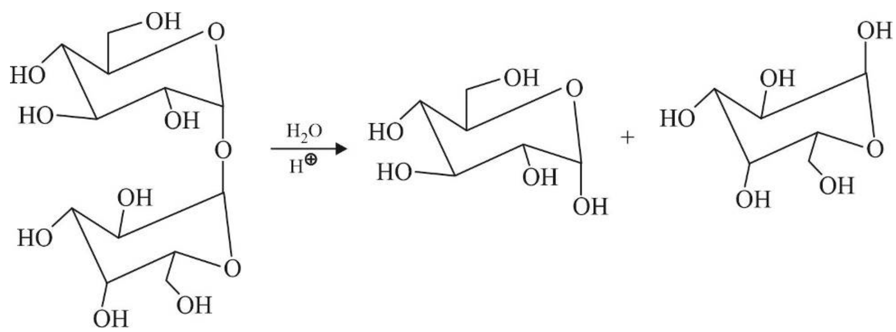
so molecular orbital configuration is $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi(2P_x)^2, \pi(2p_y)^2$

2.(D) Theoretical

3.(D)



4.(D)



Fructose doesn't get oxidized by Br_2 / H_2O. In option (C) upon acidic hydrolysis similar monosaccharides are formed.

ONE OR MORE THAN ONE CHOICE

5.(BC) $[Co(en)_3]^{+3}$ it shows optical isomerism, $[Cr(C_2O_4)_3]^{-3}$ and $[Pt(Cl_2)en_2]^{+2}$ also shows optical isomerism.

6.(ABCD) (A) $pH = 7 - \frac{1}{2}\left(pK_b + \log c\right)$

$$\Rightarrow \ 5.0 = 7 - \frac{1}{2}(4.4 + \log c)$$

$$\therefore \ c = 0.4 = \frac{80/M}{2} \ \Rightarrow \ M = 100$$

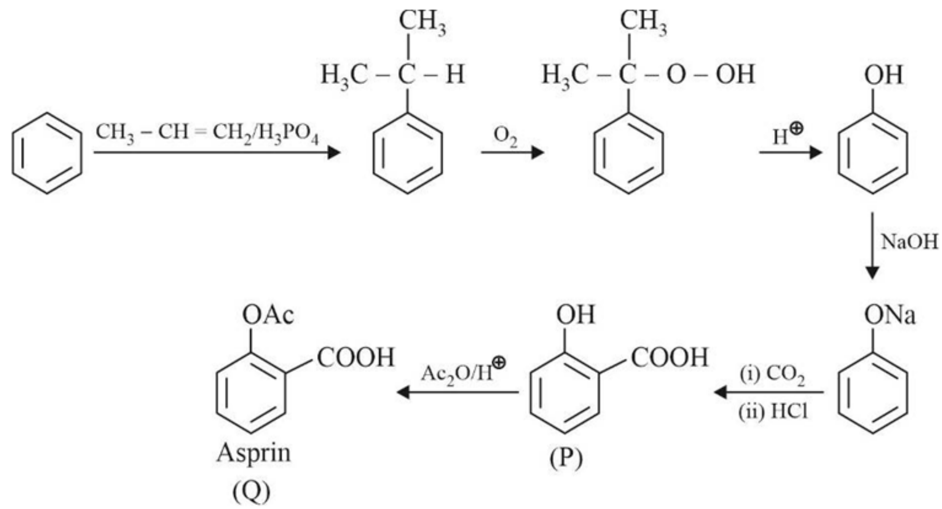
(B) $h = \sqrt{\frac{K_w}{K_b \times c}} = \sqrt{\frac{10^{-14}}{4 \times 10^{-5} \times 0.4}} = 2.5 \times 10^{-5}$

(C) $\sqrt{3}a = 2\left(r_{x^+} + r_{y^-}\right)$

$$\Rightarrow \ a = \frac{2 \times (160 + 186.4)}{\sqrt{3}} = 400\,pm$$

(D) $d = \frac{ZM}{N_A \cdot V} = \frac{1 \times 100}{6 \times 10^{23} \times \left(400 \times 10^{-10}\right)^3} = 2.6\,g/cm^3$

7.(ABD)



NON-NEGATIVEINTEGER TYPE

1.(10) $P_4 + 8SOCl_2 \longrightarrow 4PCl_3 + 2S_2Cl_2 + 4SO_2$

$x + y + z = 4 + 2 + 4 = 10$

2.(6) $SiF_4 \rightarrow sp^3$ $\quad O_3 \rightarrow sp^2$

$NF_3 \rightarrow sp^3$ $\quad SO_3 \rightarrow sp^2$

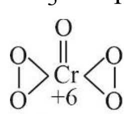
$H_2O \rightarrow sp^3$ $\quad XeO_3 \rightarrow sp^3$

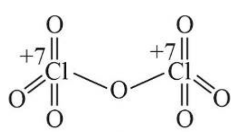
$NH_2^- \rightarrow sp^3$ $\quad [Cu(NH_3)_4]^{+2} \rightarrow dsp^2$

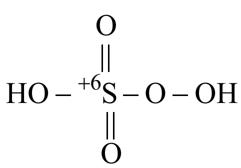
$BeH_2 \rightarrow sp$ $\quad N(SiH_3)_3 \rightarrow sp^2$

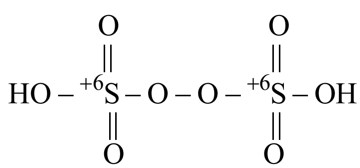
$BF_3 \rightarrow sp^2$ $\quad SiO_2 \rightarrow sp^3$

3.(4)


(CrO_5)


(Cl_2O_7)


(H_2SO_5)

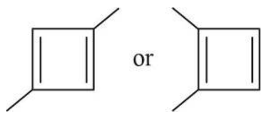

$(H_2S_2O_8)$

4.(3) $\lambda_1 - \lambda_2 = \dfrac{88}{135R}$

$$\frac{1}{RZ^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right]} - \frac{1}{RZ^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right]} = \frac{88}{135R}$$

$$\frac{88}{15RZ^2} = \frac{88}{135R} \Rightarrow Z^2 = 9 \Rightarrow Z = 3$$

5.(3) $\pi = \dfrac{\pi_1 V_1 + \pi_2 V_2}{(V_1 + V_2)} = \dfrac{2.4 \times 2V + 4.2 \times V}{2V + V} = 3 \text{ atm}$

6.(3) 

$\xrightarrow[(CH_3)_2S]{O_3} H_3C - \overset{\overset{O}{||}}{C} - \overset{\overset{O}{||}}{C} - H \xrightarrow{\text{Tollen's reagent}} \underset{[C]}{H_3C - \overset{\overset{O}{||}}{C} - \overset{\overset{O}{||}}{C} - OH}$

$\xrightarrow{NaBH_4} \underset{[D]}{H_3C - \underset{\underset{OH}{|}}{CH} - \underset{\underset{O}{||}}{C} - OH}$

$\Rightarrow M_0 = 90$

COMPREHENSION WITH NUMERICAL TYPE

7.(0.80) $\Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 + \Delta S_5 + \Delta S_6 = 0$ (because the process is cyclic)

Also, $\Delta S_2 = \Delta S_4 = \Delta S_6 = 0$ (reversible adiabatic process)

So, $\Delta S_1 + \Delta S_3 + \Delta S_5 = 0 = x$

So, $\dfrac{x+4}{5} = \dfrac{0+4}{5} = 0.80$

8.(25) $q_{total} + w_{total} = \Delta U = 0$ (cyclic process)

$q_{total} + (-700) = 0$

$\therefore\ q_{total} = 700\,J$

Now, $Q_1 + Q_2 + Q_5 + Q_2 + Q_4 + Q_6 = 700$

$500 + 800 + Q_5 + 0 + 0 + 0 = 700$

$Q_5 = -600\,J$

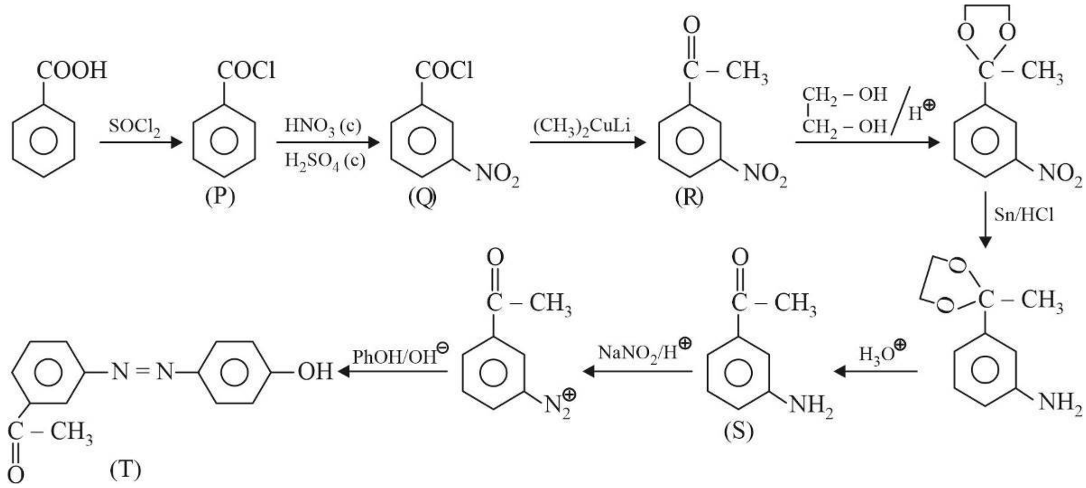
$\Delta S_1 + \Delta S_3 + \Delta S_5 + \Delta S_2 + \Delta S_4 + \Delta S_6 = 0$ (cyclic process)

$\dfrac{500}{250} + \dfrac{800}{200} - \dfrac{600}{T_5} + 0 + 0 + 0 = 0$

$\Rightarrow T_5 = 100$

$T_1 = 250K \Rightarrow \dfrac{T_1}{T_5} = 2.5$

9.(8) & 10.(135)


COOH
$SOCl_2$
COCl
(P)
HNO_3 (c)
H_2SO_4 (c)
COCl
NO_2
(Q)
$(CH_3)_2CuLi$
$C - CH_3$
NO_2
(R)
$CH_2 - OH$
$CH_2 - OH$
$H^{\oplus}$
$C - CH_3$
NO_2
Sn/HCl
$C - CH_3$
NH_2
$H_3O^{\oplus}$
$C - CH_3$
NH_2
(S)
$NaNO_2/H^{\oplus}$
$C - CH_3$
$N_2^{\oplus}$
$PhOH/OH^{\ominus}$
N = N
OH
$C - CH_3$
(T)